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# DCW-003-1161004

Seat No. ______

## M. Sc. (Sem. I) Examination

**August - 2022**

## Mathematics : CMT - 1004

*(Theory of Ordinary Differential Equation)*

**Faculty Code : 003**

**Subject Code : 1161004**

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

**Instructions :** (1) Attempt any five questions from the following.
(2) There are total ten questions.
(3) Each question carries equal marks.

1 Answer the following : **7×2=14**

(1) Define linear differential equation and linear homogeneous differential equation with an example.

(2) Write the differential equation $y_1' = y_2 + \cos t, y_2' = y_1$ in the matrix form.

(3) Find the general solution of $y''' + 3y'' + 3y' + y = 0$ on $\mathbb{R}$.

(4) Define :

(a) Wronskin.

(b) Regular Singular Point.

(5) Define Laplace transform of a function in $\mathcal{H}$ and show that, it converges absolutely.

(6) State, shifting property of Laplace transform for $z \in \mathbb{C}$.

(7) Let $A$ and $B$ be $n \times n$ matrix and $AB = BA$ then prove that, $\exp(A+B) = \exp(A) \cdot \exp(B)$.

2 Answer the following : **7×2=14**

(1) Define degree of a differential equation and linear differential equation with examples.

(2) Find two linearly independent solutions of $y'' - y = 0$.

(3) State, the first fundamental theorem of calculus.

(4) Locate and classify the singularities of

$(t-1)^3 y'' + 2(t-1)^2 y' - 7ty = 0$.

(5) Define power series and Bessel's function.

(6) Determine the largest interval of existence of the solution of the IVP for the equation : $y''' + (t^2 - 1) y = 0$ with $y(-1) = 1, y'(-1) = 0, y''(-1) = -1$.

(7) Find the linearly independent solutions of $y'' + y = 0$ on $\mathbb{R}$.

3 Answer the following : **2×7=14**

(1) State, the condition of the solution of an initial value problem of a system of 1st order linear differential equation.

(2) Solve initial value problem

$$y_1' = y_1 + y_2 + f(t), y_2' = y_1 + y_2 \text{ with } \begin{bmatrix} y_1(t_0) \\ y_2(t_0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

where $f$ is a continuous function from $I$ to $\mathbb{R}$, $t_0 \in I$.

4 Answer the following : **2×7=14**

(1) (a) Find $L^{-1}\left(\frac{3z+7}{z^2 - 2z - 3}\right)$.

(b) Find $L(\sin ct)$.

(2) Solve $y''' - 3y' + 2y = 4e^{2t}$ with $y(0) = 3$ and $y'(0) = 5$ using Laplace transform.

5 Answer the following : **2×7=14**

(1) Find the eigenvalues and eigenvectors of matrix

$$A = \begin{bmatrix} 2 & -3 & 3 \\ 4 & -5 & 3 \\ 4 & -4 & 2 \end{bmatrix}.$$

(2) Prove that, the eigenvectors corresponding to the distinct eigenvalues of $n \times n$ matrix $A$ are linearly independent in $\mathbb{R}^n$ or $\mathbb{C}^n$.

6 Answer the following : **2×7=14**

(1) Justify whether the Legendre's equation

$\left(1-t^2\right)y'' - 2ty' + n(n+1)y = 0$; (where $n$ is constant)

has a solution or not.

(2) Let $A$ be the constant $2 \times 2$ complex matrix then prove that, there exists a constant $2 \times 2$ non-singular matrix $T$ such that $T^{-1}AT$ has the form $\begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$.

7 Answer the following : **2×7=14**

(1) Find the particular solution of $y'' + y = \tan t$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; $y(0) = 0$ and $y'(0) = 0$.

(2) Define Legendre's polynomial and compute 1st, 2nd, 3rd, 4th and 5th degree.

8 Answer the following : **2×7=14**

(1) State and prove, Gronwall's inequality.

(2) Define convolution. Further show that, if $f \in \mathcal{H}$ and $\frac{f(t)}{t} \in \mathcal{H}$ then $L\left(\frac{f(t)}{t}\right) = \int_z^{\infty} L(f(w))\,dw$ for which $\operatorname{Im}(w)$ is bounded and $\operatorname{Re}(w) \to \infty$.

**9** Answer the following : **2×7=14**

(1) State and prove, Abel's theorem.

(2) Find the fundamental matrix of $y' = A(t)y$ on $(-\infty, \infty)$, where $A(t) = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \forall\, t \in (-\infty, \infty)$ and find $\exp(tA); \forall\, t \in (-\infty, \infty)$.

**10** Answer the following : **2×7=14**

(1) (a) Define second shifting theorem.

(b) Find $L\left(e^{-t}\right)(z)$ using definition of Laplace transform.

(2) Prove that, if $a_0(t), a_1(t), a_2(t)$ which are analytic at $t_0$ and $t_0$ is a regular singular point of $a_0(t)y'' + a_1(t)y' + a_2(t) = 0$ then given equation can be written in the form $(t - t_0)^2 y'' + (t - t_0)\alpha(t)y' + \beta(t)y = 0$ for some function $\alpha(t)$ and $\beta(t)$, which are analytic at $t_0$ and not all $\alpha(t_0), \beta(t_0)$ and $\beta'(t_0)$ are zero.

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